An Introduction to Topological Insulators

Topological insulators embody a new quantum state of matter. To put this into context, let us quickly look back at the history of quantum mechanics. Planck’s law was derived in 1900, Bohr’s atomic model was introduced in 1913, the Schrödinger equation was formulated in 1926, and Dirac’s elegant relativistic equation completed the initial development of quantum mechanics in 1928. With over a century of history, one would expect that nothing fundamental is left to be discovered in quantum mechanics. Yet, we are now discovering a new fundamental aspect – and topological insulators are where the action is taking place.

This new development started in 1980, when von Klitzing [1] discovered the quantum Hall effect. It was recognized in 1982 by Thouless et al [2] that this amazing phenomenon is due to the quantum mechanical wavefunction of the quantum Hall system having a nontrivial topology in Hilbert space. In many cases, the structure of the Hilbert space spanned by the wavefunction of a given system is simple, or ‘trivial’. However, it is possible that the Hilbert space takes on a ‘nontrivial’ topology. In algebraic topology, a torus is topologically distinct from a sphere; it is characterized by a non-zero topological number (genus 1) and cannot be continuously deformed into a sphere which is topologically trivial (genus 0). Similarly, the Hilbert space of the quantum Hall system has a distinct topological structure and the so-called Chern number characterizes its nontrivial topology.

In 2005 Kane and Mele [3] discovered that the Hilbert space of even a simple band insulator can be topologically nontrivial: in this case, the so-called $Z_2$ invariant characterizes the nontrivial topology. In group theory, the group of integer numbers is called $Z$, and its quotient group classifying even and odd numbers is called $Z_2$; hence, the $Z_2$ invariant distinguishes even and odd. In grossly oversimplified terms, the parity of the wavefunction of a topological insulator is odd, while the parity of the vacuum wavefunction is even, and it follows that topological insulators and the vacuum are distinct in the $Z_2$ topology. A common property of topological insulators and the vacuum is the existence of an excitation gap which protects the $Z_2$ invariant from perturbations. Hence, their distinct $Z_2$ topology dictates that the gap must close at the interface for the $Z_2$ invariant to change, which means that a gapless metallic state necessarily arises at the edge or surface of a topological insulator facing a vacuum. Therefore, topological insulators can be considered as a new quantum state of matter characterized by an intrinsically metallic edge/surface on top of an insulating bulk.
After the $\mathbb{Z}_2$ topology was discovered, remarkably rapid developments followed: theoretical predictions for concrete realizations of two- and three-dimensional topological insulators were made by Bernevig et al. [4], and by Fu and Kane [5], respectively, and they were soon experimentally verified by groups at Würzburg [6] and Princeton [7], respectively. About ten materials have already been experimentally confirmed to be topological insulators. So far, all those materials share a common trait: that is, a strong spin–orbit coupling causes band inversion and flips the parity of the valence band. Coincidentally, the nature of the edge/surface state of a topological insulator is dictated by the spin–orbit coupling so that the spin direction is perpendicularly ‘locked’ to the momentum, forming the so-called helical spin polarization. Interestingly, such a helical spin polarization means that there exists a dissipationless spin current at the edge/surface of a topological insulator in equilibrium, and this carries tantalizing implications for spintronics.

Intriguingly, it has been predicted by Fu and Kane [8] that if superconductivity is induced via the proximity effect on the surface of a 3D topological insulator, the surface hosts a non-Abelian Majorana zero-mode in the vortex, which is the essential ingredient for realizing fault-tolerant topological quantum computing. It has been shown by Qi et al. [9] that the quantum field theory of topological insulators resembles that of the fictitious ‘axion’ particle, and this theory leads to various interesting predictions such as quantization of the magnetoelectric effect, the appearance of an image magnetic monopole, and the half-integer quantum Hall effect. All these phenomena, yet to be experimentally discovered, will deepen our understanding of quantum mechanics in a fundamental way.

Obviously, topological insulators offer a fertile ground for many theoretical and experimental discoveries. But we are just about to uncover a new arena of quantum mechanics enriched by the nontrivial topology of the Hilbert space. For example, the superconducting wavefunction can also have nontrivial topology, and the physics of such ‘topological superconductors’ is largely unexplored, particularly experimentally. EPL is determined to serve the community by publishing timely, high-quality papers in the exciting field of topological insulators.

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