Chaos-order transition in the hot electrons system under Gunn effect, observed with a new method of two-dimensional representation of phase portraits

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Abstract
A new method was proposed of two-dimensional representation of phase portraits for investigations of dynamics of non-linear systems. The method was successfully applied to the system of non-equilibrium carriers in semiconductor under photo-induced Gunn effect and can be recommended to express-study of evolution processes in other self-organizing systems, as it allows to represent easily the phase portrait of any dimension as two-dimensional curve and to define the number of its loops and hence the chaotization level of the system.

Introduction
Modern science pays significant attention to non-linear and non-equilibrium many-particle systems, which tend to self-ordering or self-structurization under given external influence [1]. Exact analytical solutions for the systems named are very complicated and in general are impossible to find; for that one needs to use numerical methods to solve the system of non-linear differential equations that describes the system and to determine the stability of the solution obtained. Distribution of the phase variables in time or space is usually represented as n-dimensional curve in phase space, called phase portrait of the system. Its chaotization level reflects the instability of the system and can be characterized using different quantities, such as Lyapunov exponents, Hausdorff dimension, power spectral densities of components, etc [1-3]. All the given methods are rather complicated and therefore have comparably large calculation time; moreover, only some of them characterize the entire phase
portrait, but the majority is focused to derive the necessary values from single phase variable
dependence. Therefore it is necessary to find fast and simple express methods to define the system
chaotization level and to characterize the whole phase portrait. In the given paper we have proposed
new fast method to determine the amount of loops of the phase portrait and hence its complexity,
projecting adequately n-dimensional phase portraits into plane of two dimensions using modification of
trajectory tracing method developed before [4].

The method proposed was tested on the self-organized system of carriers in semiconductor
under specific illumination conditions, when according to [5] new non-linear optical effect of multiple
high-field domain formation takes place; the illumination should be performed with two waves with
small frequency difference that forms moving interference pattern on the surface of the crystal. The
domains excited are moving through the sample phase-locked with illumination pattern, introducing
periodic modulation of refraction coefficient via Pockelce effect. Devices working on the phenomena
described, referred as photo-induced Gunn effect, are of great scientific interest because of their
possible application for direct transformation of optical signals into electric ones, quick optical
switches, real-time polarization holograms and sensitive low-latency optical detectors.

Theoretical model

To investigate the chaotization level of the non-linear system with changes of external control
parameters we have proposed method of two-dimensional representation of the phase portraits, which
is one of the possible developments of the trajectory tracing method [4]. The information on the phase
portrait topology could be represented as a sequence of distances $R_i$ from the given point with number
$M$ that belongs to the attractor to the any point of the phase portrait. When the phase point returns to
the point topologically equivalent to $M$ on the phase curve, the measured distance becomes equal to
zero. The loops of the phase portrait hence correspond to different extremum values of the distance $R$,
allowing us to characterize the phase portrait by its unique characteristic sequence of extrema and
distances between them over the independent variable. To make the detection of the extremum values
more explicit we have proposed to build the two-dimensional representation of the phase portrait in
$RR'$ space, i.e. as a set of points with coordinates $R_i$ and corresponding derivatives $R_i'$, where the latter
are equal to zero for extremum values of $R_i$. Counting amount of topologically different intersection of
the curve obtained with $R'\geq0$ line, one can obtain the information about the number of loops of the
phase portrait, as each loop introduces one maximum and one minimum value to $R_i$ distribution. When
the topology of the phase portrait changes due to the bifurcations of different kind, the $RR'$ image
changes corresponding, introducing new loops as it takes place to the phase portrait.
The method proposed was applied to find the chaotization level of non-equilibrium carrier system in semiconductor sample illuminated with two waves with slightly different frequencies that have the intensity [5]

\[ I(z,t) = I_0 \left[ 1 + m \cos (kz + \Omega t) \right]. \quad (1) \]

In (1) \( k \) is the wave number and \( m \) is modulation depth; the value \( I_0 \) is intensity of the incident light averaged over the time. Optical excitation of the carriers takes place with different generation rates due to different light intensities, thus forming domains moving phase-locked with the interference pattern created by illumination. To describe the carrier dynamics of the system one can use one-dimensional model, which for the case of dimensionless variables has a form [5]:

\[
\begin{align*}
\frac{\partial y_1}{\partial \tau} &= a f(x, \tau)(b - y_1) - y_1 y_2 \\
\frac{\partial y_2}{\partial \tau} &= \frac{\partial y_1}{\partial x} + \alpha \frac{\partial}{\partial x} \left[ y_2 v(y_3) + \beta \frac{\partial y_2}{\partial x} \right] \\
\frac{\partial y_3}{\partial x} &= -\frac{1}{\alpha \beta} (y_2 - y_1 + 1)
\end{align*}
\]

where

\[
\tau = \gamma N_A t, \quad x = \varepsilon \varepsilon_0 E_s y z / e D, \quad y_1 = N_D^i / N_A, \quad y_2 = n / N_A, \quad y_3 = E / E_s,
\]

\[
a = s \varepsilon_0 \gamma N_A, \quad b = N_D / N_A, \quad \alpha = \varepsilon \varepsilon_0 E_s v_i / e D N_A, \quad \beta = \varepsilon \varepsilon_0 E_s v / e v_s, \quad k = e D \gamma / e \varepsilon_0 E_s y, \quad \omega = \Omega / \gamma N_A, \quad v = v(y_3) v_s = y_3 (1 + Ay_3^3) / (1 + Ay_3^4), \quad f(x, \tau) = 1 + m \cos (kx + \omega \tau).
\]

In (2) and (3) \( N_D \) and \( N_D^i \) are the concentrations of donors and ionized donors, \( N_A \) is concentration of ionized acceptors, \( s \) is photo-ionization cross-section, \( \gamma \) and \( D \) are coefficients of recombination and diffusion, \( E_s \) designates saturation field and parameter \( A \) defines the drift velocity of electrons \( v_i \); other designations are common. For the stationary inhomogeneous case when \( \tau \rightarrow \infty \) the system (2) could be significantly simplified to:

\[
\begin{align*}
\frac{\partial y_2}{\partial x} &= \frac{1}{\beta} \left[ J_c - y_2 v(y_3) \right] \\
\frac{\partial y_3}{\partial x} &= -\frac{1}{\alpha \beta} \left[ 1 + y_2 - \frac{a b (1 + m \cos k x)}{y_2 + a (1 + m \cos k x)} \right]
\end{align*}
\]

(4)
The value $J_c$ in (4) is the density of electric current in the external circuit that is a function of time but not depends on the coordinate.

**Results and discussion**

Solutions of the system (4) were found by numerical integration using Runge-Kutta method of the fourth degree. Obtained distributions of phase variables over the coordinate $x$ forms two-dimensional phase portraits in $y_2y_3 (n-E)$ phase space, which are limit cycles with different amount of loops depending on the value of the control parameter. We have chosen the incident light intensity $I_0$ to be the control parameter because has significant influence on the behavior of the system [6].

Two characteristic phase portraits with corresponding $RR'$ representations are shown in Fig.1 and Fig.2. As one can see in Fig.1, the amount of loops of the phase portrait is two, and $RR'$ curve correspondingly have four different intersections of the derivative $R'_i$ with $R'=0$. For the case shown in Fig.2 when the phase portrait has three loops, $RR'(i)$ curve have six different points where $R'=0$. As the amount of intersections could be easily defined by computer, we have calculated the dependence of loop number $L$ over the intensity of the illumination $I_0$ (Fig. 3). The evolution of system studied over this control parameter is characterized by the intermittency of two topologically different phase portraits [6], resulting in different oscillation profiles. As one can see there exist five distinct regions of control parameter values when system is being characterized by phase portraits with two loops (Fig. 3, region A and C), three loops (Fig. 3, region B and D) and one single loop (Fig. 3, region E). Each of the named regions has its own oscillation type of the phase variables; the characteristic phase portraits for two- and three-loop configuration are shown in Fig. 1 and Fig. 2. As behavior of the system has strong dependence over $I_0$, even small changes of control parameter near the bifurcation points could change the oscillation regime of the phase variables. In this way using the data presented in Fig. 3 the system could be controllable switched to the necessary oscillation regime using the appropriate values of $I_0$.

**Conclusions**

The proposed method of two-dimensional representation of phase portraits in $RR'$ plane is simple and easy to implement and in the same time it has sufficient precision to determine the bifurcation points of the system, where the state topology switching occurs. The method was successfully tested for non-equilibrium systems of non-equilibrium carriers under photo-induced Gunn effect; it could be recommended for investigation of the evolution of different self-organization systems, defining phase portrait complexity level by simple intersection counting procedure processing unified two-dimensional curve for any arbitrary quantity of phase variables of the system studied.
Acknowledgement

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REFERENCES

FIGURE CAPTIONS

FIG.1. Phase portrait of the system studied (a) for \( I_0 = 6.2 \times 10^{11} \) W/m\(^2\) and corresponding \( RR' \) curve (b). Circles denote intersections of \( RR' \) curve with \( R' = 0 \).

FIG.2. Phase portrait of the system studied (a) for \( I_0 = 7.2 \times 10^{11} \) W/m\(^2\) and corresponding \( RR' \) curve (b). Circles denote intersections of \( RR' \) curve with \( R' = 0 \).

FIG.3. Dependence of the loop number \( L \) of the phase portrait on the incident light intensity \( I_0 \).
FIG. 1
FIG. 2
FIG. 3